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Optimal Servicing of Geosynchronous Satellites

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I. Introduction

THE robotic servicing of satellites in orbit is a concept that is receiving increasing attention. If this could be achieved economically, then the system lifetime of many satellites could be extended and system costs reduced. The best economy will be achieved when numerous satellites can be serviced in one servicing mission. Because plane changes require a lot of fuel, two scenarios in which numerous satellites could be serviced are constellations with many satellites in each plane, such as Iridium and global positioning system satellites, and geosynchronous satellites. A few satellites could potentially be serviced in low Earth orbit using differential nodal precession if all the satellites had approximately the same inclination.

Even though there is now a policy that satellites are to be removed from the geosynchronous belt at end of life, there are many nonoperational satellites in geosynchronous orbit that are a hazard to other satellites. If there was a way to rendezvous with these satellites and attach a small engine to raise their orbit, then over a period of time the geosynchronous belt could be made safer.

The method developed in this Note determines the minimum fuel solution for visiting (rendezvousing with) a set of satellites in geosynchronous orbit with small inclinations. Because the fuel to rendezvous with a satellite in the same orbit is very small if sufficient time is allowed for the maneuver, the method focuses on finding the order in which the satellites should be visited to minimize the fuel required for the plane changes. It is shown that the minimum fuel solution is proportional to the minimum distance path through the set of points that are the projections of the angular momentum vectors on the equatorial plane. Thus, the minimum fuel (Δv) solution is the solution of the traveling salesman problem (TSP). The fuel required for the in-plane maneuver is not minimized for several reasons. It is much smaller than the fuel required for the plane change, and in many cases it can be obtained by directing the out-of-plane (plane change) Δv to have a small in-plane component. In this way the required in-plane Δv is obtained with only a very small increase in total Δv. Strategies for the rendezvous include equal time for each rendezvous or equal Δv for the in-plane portion of the maneuver. Parameters in the problem are the total time and the servicing time at each satellite.

II. Analysis

Consider a set S of N geosynchronous satellites, where S_i represents the ith satellite with orbital elements $\boldsymbol{e}_i = (e_{i1}, e_{i2}, \ldots, e_{i6})^T$. The objective is to find the order of visiting the satellites such that the total fuel (Δv) to rendezvous (visit) is minimized. In addition, we want to identify those satellites that should not be serviced because the fuel to service them is prohibitive. In this preliminary study we restrict the set S to consist of satellites in circular orbits and equal periods of 24 h, arbitrary right ascension, and small, but not necessarily zero, inclination. The circular orbit and 24-h restrictions could be relaxed to consider near circular and near 24-h satellites, that is, those that drift around the geosynchronous belt. First we show that the solution for the order of visiting each of the satellites that results in the minimum Δv solution for the plane-change portion of the Δv to rendezvous with each of the N satellites in S is the solution of the TSP.

Consider two satellites of the set S with inclination and right ascensions (I_1, Ω_1) and (I_2, Ω_2). Let (X, Y, Z) be an inertial coordinate system with Z along the polar axis, and X and Y define the equatorial plane. As shown in Fig. 1, the coordinates of the projection of the angular-momentum vector in the equatorial plane are $h_j \sin I_j(\sin\Omega_j, -\cos\Omega_j)$, $j = 1, 2$. With equal angular momentum, the distance d between the two projected vectors is

$$d^2 = h^2\big[(\sin I_2 \sin\Omega_2 - \sin I_1 \sin\Omega_1)^2 + (\sin I_2 \cos\Omega_2 - \sin I_1 \cos\Omega_1)^2\big]$$

$$d^2 = h^2\big\{(\sin I_2 - \sin I_1)^2 + 4\sin I_2 \sin I_1 \sin^2[(\Omega_2 - \Omega_1)/2]\big\} \quad (1)$$

For small inclinations this becomes

$$d^2 = h^2\big[(I_2 - I_1)^2 + 4I_1 I_2 \sin^2(\Delta\Omega/2)\big] \quad (2)$$

Let the angle between the two orbit planes be γ, which is given by

$$\cos\gamma = \cos I_1 \cos I_2 + \sin I_1 \sin I_2 \cos\Delta\Omega \quad (3)$$

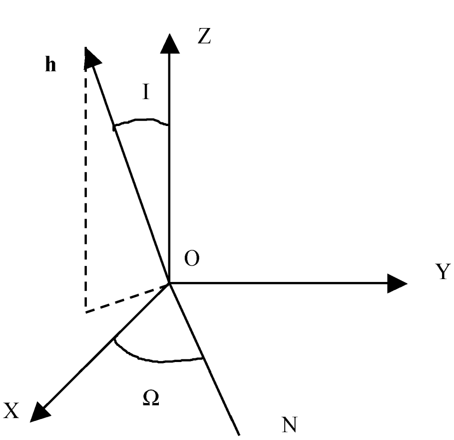

Fig. 1 Angular momentum projection.

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For small inclinations this becomes

$$\gamma^2 = (I_2 - I_1)^2 + 4I_1I_2\sin^2(\Delta\Omega/2) = (d/h)^2 \tag{4}$$

Thus, the angle between the planes is proportional to the distance between the projections of the angular-momentum vectors in the equatorial plane. The Δv for a plane change of γ is

$$\Delta v = 2v\sin(\gamma/2) \approx v\gamma \tag{5}$$

Thus, the Δv for the plane change is proportional to the distance between the projections of the angular momentum vectors on the horizontal plane.

The total Δv consists of two components, the plane change and the in-plane transfer for the rendezvous. Because the satellites are in circular orbits of the same period, the in-plane transfer is just an impulse into a small eccentric orbit to change the period to accomplish an orbit phase change, followed by a circularization maneuver. If sufficient time is allowed this Δv will be much smaller than the plane change. It may also be possible to combine it with the plane change, which would help to minimize the total mission Δv. In either case, the plane change Δv dominates the total Δv. Thus, the minimum Δv is found by minimizing the Δv for the plane change.

Now consider a set of geosynchronous satellites that have small inclinations. The servicing satellite is referred to as the interceptor and the satellite it is going to rendezvous with is the target. Because the Δv for each plane change is proportional to the distance between the projections of the angular-momentum vectors on the equatorial plane, the minimum Δv for the plane changes required to visit all the satellites is the minimum path distance through all the projections of the angular-momentum vectors on the equatorial plane. This is the TSP, a problem that has been studied extensively. Any point that is far from the other points is one that requires substantial Δv to reach and can be eliminated from consideration. Because visiting many satellites could require several years, one also has to consider the effect of the gravitational perturbations that will result in the projections of the angular-momentum vectors slowly changing with time. This makes the problem a dynamic TSP (DTSP). In this initial study we do not consider the DTSP.

A. In-Plane Transfer

The in-plane portion of the rendezvous is accomplished by a tangential Δv to change the mean motion of the interceptor and letting the two satellites drift until the longitudes match. Let $\lambda_{j,k}$ be the longitude separation between satellites j and k, and let the time for the interceptor to travel from satellite j to satellite k be $\tau_{j,k}$. To travel the shortest angular distance, we transfer the interceptor to higher orbit to slow down or transfer to lower orbit to speed up. Therefore, $|\lambda_{j,k}| \leq \pi$. Because the radii of the transfer and target orbit match every orbit, $\tau_{j,k}$ will have to be an integral number of orbits, that is,

$$\tau_{j,k} = N_{j,k}T_g \tag{6}$$

where T_g is the geosynchronous orbit period. We also have

$$|\lambda_{j,k}| = |\Delta n|\tau_{j,k} = |\Delta n|N_{j,k}T_g \tag{7}$$

From the energy integral and the mean motion equation, we get

$$\Delta n = -(3n/2)(\Delta a/a) = -3n(\Delta v/v) \tag{8}$$

By substituting in Eq. (6), we get

$$\Delta v N_{j,k} = (R_g/3T_g)|\lambda_{j,k}| \tag{9}$$

where R_g is the geosynchronous radius. Two possible strategies are to select the same time to rendezvous for each pair of satellites or to allow the same amount of Δv for the in-plane transfer for each pair of satellites. In either case, as long as the time to rendezvous is not too small, the in-plane Δv should be much smaller than the Δv required for the plane change, and we can obtain the in-plane portion for essentially no cost by redirecting the out-of-plane thrust to obtain the required tangential portion.

In this analysis we do not consider the time or fuel required for the terminal phase of rendezvous.

B. Traveling Salesman Problem

Given a set of points, the TSP consists of finding the path of minimum length that passes through each of the points. The DTSP is the TSP when each of the points follows a prescribed path. References 1–3 present some methods for its solution. There is no closed-form solution, and all methods are numerical.

III. Results

To demonstrate the method, the 20 objects in geosynchronous orbit shown in Table 1 were selected. The inclination and right ascension are those from the Space Object catalog. To demonstrate that the visitation order is not to proceed sequentially around the geosynchronous belt, the longitude was distributed randomly from 0 to 360 deg. It is assumed that all the objects are in circular orbits.

Figure 2 shows the projections of the angular-momentum vectors and the minimum-distance path through the points. It is evident that

Table 1 Selected satellites with properties

Object	Name	Inclination, deg	Rt. ascension, deg	Longitude,°
16274	Morelos 2	0.9128	90.595	197.63
19772	Intelsat VA F15	1.7156	87.1401	221.59
19883	TDRS 4	2.3935	82.265	68.81
15677	Gstar 1	2.6347	80.6943	169.65
22316	IUS-13 SRM-2	4.1882	43.4184	130.83
21641	IUS-15 SRM-2	6.3096	70.5983	85.52
13595	Intelsat V F5	7.5253	49.9744	277.91
12089	Intelsat V F2	8.2849	46.1791	96.56
12474	Intelsat 501	8.7881	44.4057	28.91
12472	GOES 5	9.9086	39.2166	354.07
13636	DSCS II F-16	10.0680	41.2597	146.77
13643	IUS-2 SRM-2	11.2379	37.5653	287.66
15236	Leasat 2	11.8924	40.1735	168.11
12046	FLTSATCOM F4	12.2709	31.7337	28.04
11621	DSCS II F-13	12.6462	33.5862	127.88
11144	FLTSATCOM F5	13.0725	55.3282	81.67
10669	DSCS II F-11	13.0939	32.6799	191.45
10000	FLTSATCOM F1	14.0176	26.7312	274.49
11256	SCATHA	14.4197	43.9506	309.11
12635	DSCS II F-7	15.2555	25.2461	203.62

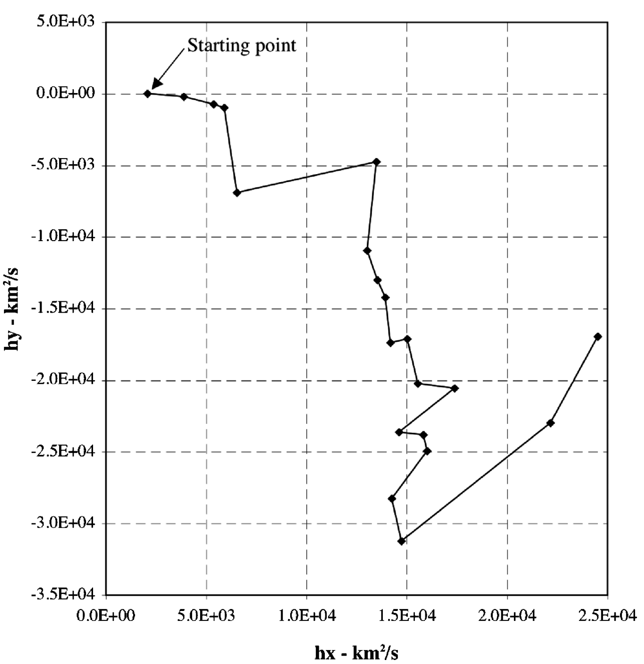

Fig. 2 Minimum Δv solution.

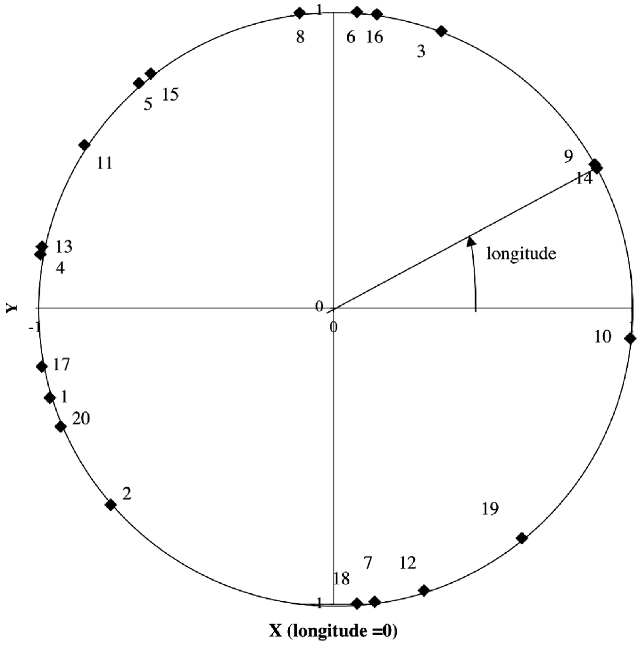

Fig. 3 Longitude distribution and visitation order.

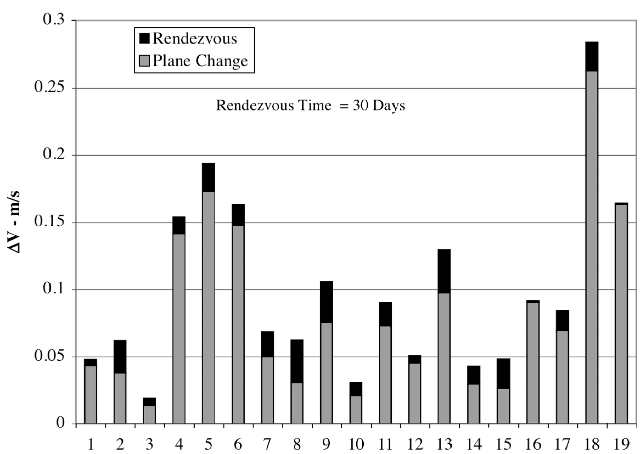

Fig. 4 Total Δv for minimum Δv solution, 30-day transfer time.

much of the minimum path can be visually determined. Although not presented here, we selected several sets of objects from the Space Object catalog and we found that in each case we could visually determine much of the minimum-distance path. Because much of the solution can be visually determined, the remainder of the solution was obtained by trial and error. Figure 3 shows the order of the objects visited and demonstrates that the optimum solution is not to progress sequentially around the geosynchronous belt. Figure 4 shows the total Δv along with the amount for the plane change and that required for rendezvous or the time to transfer from one satellite to the next. For this calculation they have been added, although as pointed out earlier that probably is not necessary. In Fig. 4 the rendezvous or transfer time is 30 days. The high cost of visiting the group of three in the middle and the last two objects is evident. From Fig. 2 we can also identify the objects that will be costly to visit. Also shown in Fig. 2, and not revealed by Fig. 4, is the effect of not visiting one of the objects. If the second object of the group of three near the beginning is eliminated, then it is evident from Fig. 2 that the next Δv will decrease.

Figure 5 shows the impact of increasing the service time, which decreases the time allowed for transfer. A mission time $T_m = 5$ years was selected and the servicing time T_s is a parameter. The total time allowed for rendezvous is then

$$T^* = T_m - NT_s \tag{10}$$

Two strategies for rendezvous were used. The first allows the same time for rendezvous or transfer for each rendezvous, which means the transfer Δv will vary according to the longitude difference $\lambda_{j,k}$ between the satellites. The transfer time is

$$\tau_{j,j+1} = T^*/(N-1), \qquad N = 20, \qquad j = 1, N-1 \tag{11}$$

The second strategy allows approximately the same Δv for each transfer. Because the Δv is approximately linear with the required longitude change, the transfer time for each transfer is

$$\lambda^* = \sum_{j=1}^{N-1} \lambda_{j,j+1}$$

$$\tau_{j,j+1} = T^* \frac{\lambda_{j,j+1}}{\lambda^*}, \qquad j = 1, N-1 \tag{12}$$

One sees that the solution is relatively insensitive to the servicing time until the transfer time is short enough that the Δv for the transfer begins to dominate. Both strategies result in essentially the same total Δv.

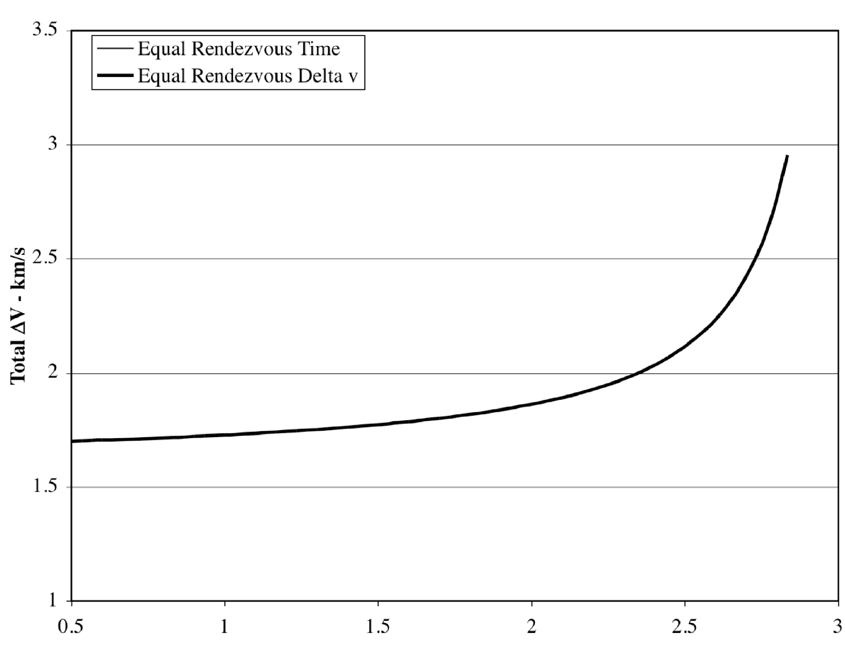

Fig. 5 Total Δv as a function of service time.

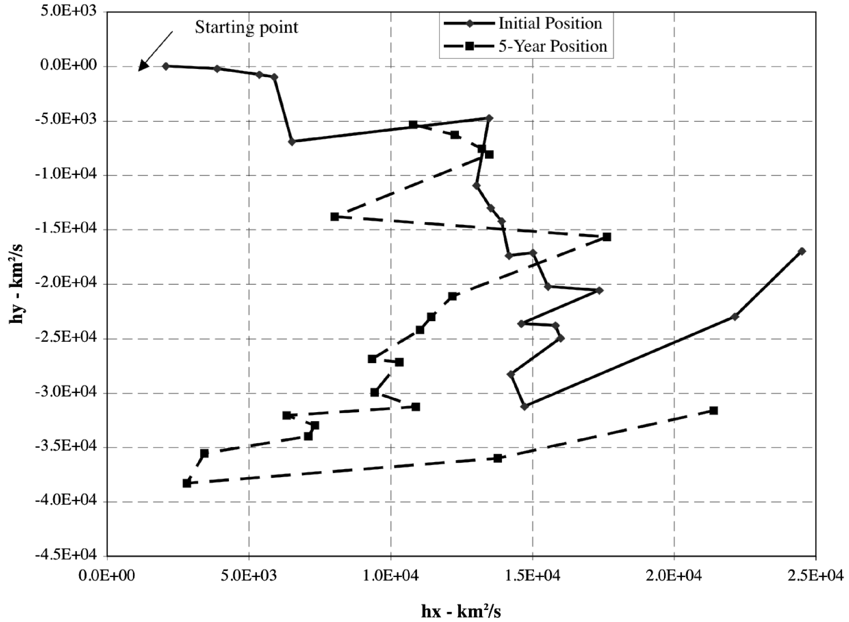

Fig. 6 Five-year time history of angular momentum projection.

One approach for servicing the satellites would be to visit them sequentially around the geosynchronous belt. Allowing 30 days for each transfer, the total Δv was 8.26 km/s, approximately four times greater. Thus, the solution obtained from the method developed in this Note results in substantial savings.

As mentioned earlier, it was assumed that the right ascension and inclination were constant. Because they vary slowly with time, it will be necessary to solve a DTSP. Figure 6 shows the angular-momentum projections at the beginning and the end of the 5-year period. The path through the points at 5 years is the same order as the initial path so that correspondence can be made between the points at the initial and final times. It appears that the order of visitation for the path of minimum distance does not change. However, this has not been verified.

IV. Conclusions

The order for rendezvousing or visiting with a set of satellites in geosynchronous orbit with small inclinations such that the total Δv is minimized has been shown to be the path of minimum distance through the points that are the projection of the angular-momentum vector of each satellite on the equatorial plane. Finding the path of minimum distance is the traveling salesman problem. Results were presented for a set of 20 satellites from the Space Object catalog. For this set of satellites, the Δv from this method is 25% of the Δv for visiting each satellite by proceeding sequentially around the geosynchronous belt. By using this method, servicing a large number of satellites in geosynchronous orbit that have small inclinations is feasible. A final solution would have to consider the time variation of the right ascension and inclination, which would require solving a dynamic traveling salesman problem. For the set of satellites selected and the time variation of the right ascension and inclination over five years, it appears that the visitation order would not change.

References

[1]Miller, D. L., and Pekny, J. F., "Exact Solution of Large Asymmetric Traveling Salesman Problems," *Science*, Vol. 251, Feb. 1991, pp. 754–761.

[2]Huang, C. Y., and Lane, S. H., "Solution to the Traveling Salesman Problem Using Heuristic Function Maximization," *Journal of Guidance, Control, and Dynamics*, Vol. 11, No. 5, 1988, pp. 430–435.

[3]Saxena, V., Reddy, G. N., and Bean, W. C., "An Improved Neural Network Simulator for Solving the Traveling Salesman Problem," *Proceedings of the 3rd Workshop on Neural Networks: Academic/Industrial/NASA/Defense*, Auburn Univ., AL, 1992.